

Lorentz and CPT Tests in QED

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Abstract. A theoretical framework extending QED in the context of the standard model is used to analyze a variety of Lorentz and CPT tests in atomic systems. Experimental signatures of possible Lorentz and CPT violation in these systems are investigated, and bounds are discussed.

INTRODUCTION

Lorentz symmetry and CPT appear to be exact fundamental symmetries of nature. All of the known physical interactions seem to be invariant under continuous Lorentz transformations consisting of boosts and rotations and under the combined discrete symmetry CPT formed from the product of charge conjugation C, parity P, and time reversal T. These symmetries are connected by the CPT theorem [1], which states that under mild technical assumptions all local relativistic field theories of point particles are symmetric under CPT.

Many of the shapeliest tests of Lorentz and CPT symmetry have been performed in atomic systems where the predominant interactions are described by quantum electrodynamics [2,3]. For example, the Hughes-Drever type experiments [4] typically compare two clocks or high-precision magnetometers involving different atomic species. The best CPT tests for leptons and baryons cited by the Particle Data Group are made by atomic physicists using Penning traps. These experiments have obtained a bound on the g -factor difference for electrons and positrons given by [5]

$$r_g^e = \frac{|g_{e^-} - g_{e^+}|}{g_{\text{avg}}} \lesssim 2 \times 10^{-12} \quad , \quad (1)$$

while experiments with protons and antiprotons have obtained bounds on the difference in their charge-to-mass ratios given by [6]

$$r_{q/m}^p = \frac{|(q_p/m_p) - (q_{\bar{p}}/m_{\bar{p}})|}{(q/m)_{\text{avg}}} \lesssim 9 \times 10^{-11} \quad . \quad (2)$$

Similarly, two proposed experiments at CERN intend to make high-precision comparisons of the 1S-2S transitions in trapped hydrogen and antihydrogen [7]. The 1S-2S transition is a forbidden transition that can only occur as a two-photon transition. It has a long lifetime and a small relative linewidth of approximately 10^{-15} . Atomic experimentalists believe that ultimately the line center might be measured to 1 part in 10^3 yielding a CPT bound

$$r_{1S-2S}^H = \frac{|\Delta\nu_{1S-2S}|}{\nu_{1S-2S}} \lesssim 10^{-15} - 10^{-18} \quad . \quad (3)$$

The main focus of the work described here is to investigate these and other tests of Lorentz and CPT symmetry performed in QED systems. The general goals of this work have been to analyze the sensitivity of QED systems to possible Lorentz and CPT violation, to uncover possible new signals that can be tested in experiments, and to estimate attainable experimental bounds in the context of a common framework that permits comparisons across different experiments. To accomplish this, we used the standard-model extension developed by Kostelecký and collaborators [8–10] as our theoretical framework. This model permits a detailed investigation of Lorentz and CPT tests in all particle sectors of the standard model. Our analysis here focuses on the QED sector of the standard-model extension. This is presented in the following sections and is used to examine five different QED systems: experiments in Penning traps [11], clock-comparison tests [12], experiments with hydrogen and antihydrogen [13], Lorentz and CPT tests with macroscopic spin-polarized materials [14] and muon experiments [15].

As a result of our investigations, several atomic experimental groups have recently reanalyzed existing data or taken new data to obtain improved bounds on Lorentz and CPT violation. Summaries of these bounds as well as estimates of bounds that can be attained in future experiments are presented in the following sections.

STANDARD-MODEL EXTENSION

Many different ideas for violation of Lorentz or CPT symmetry have been put forward over the years since the proof of the CPT theorem. A sampling of these includes the following: nonlocal interactions [16], infinite component fields [17], a breakdown of quantum mechanics in gravity [18], Lorentz non-invariance at a fundamental level [19], spontaneous Lorentz violation [8], and CPT violation in string theory [9,20].

To explore the experimental consequences of possible Lorentz or CPT violation, a common approach is to introduce phenomenological parameters. Examples of this include the anisotropic inertial mass parameters in the model of Cocconi and Salpeter [21], the δ parameter used in kaon physics [22], and the $\text{TH}\epsilon\mu$ model which couples gravity and electromagnetism [23]. Another

approach is to introduce specific lagrangian terms that violate Lorentz or CPT symmetry [24,25]. These approaches have the advantages that they are straightforward and are largely model independent. However, they also have the disadvantages that their relation to experiments can be unclear and they can have limited predictive ability. To make further progress, one would want a consistent fundamental theory with CPT and Lorentz violation. This would permit the calculation of phenomenological parameters and the prediction of signals indicating symmetry violation. No such realistic fundamental theory is known at this time. However, a candidate extension of the standard model incorporating CPT and Lorentz violation does exist.

The standard-model extension of Kostelecký and collaborators is an effective theory based on the idea that Lorentz and CPT symmetry can be spontaneously broken in the context of a more fundamental theory [8]. It is motivated in part from string theory [9]. The idea is to assume the existence of a fundamental theory in which Lorentz and CPT symmetry hold exactly but are spontaneously broken at low energy. As in any theory with spontaneous symmetry breaking, the symmetries become hidden at low energy. The effective low-energy theory contains the standard model as well as additional terms that could arise through the symmetry breaking process. A viable realistic fundamental theory is not known at this time, though higher dimensional theories such as string or M theory are promising candidates. A mechanism for spontaneous symmetry breaking can be realized in string theory because suitable Lorentz-tensor interactions can arise which destabilize the vacuum and generate nonzero tensor vacuum expectation values.

Colladay and Kostelecký have derived the most general extension of the standard model that could arise from spontaneous Lorentz symmetry breaking of a more fundamental theory, maintains $SU(3) \times SU(2) \times U(1)$ gauge invariance, and is renormalizable [10]. They have shown that the theory maintains many of the other usual properties of the standard model besides Lorentz and CPT symmetry, such as electroweak breaking, energy-momentum conservation, the spin-statistics connection, and observer Lorentz covariance. Issues related to the stability and causality of the standard-model extension are investigated in Ref. [26]. In addition to the atomic experiments described here, the standard-model extension has been used to analyze Lorentz and CPT tests with neutral mesons [27,28], photon experiments [10,24,29], and baryogenesis [30].

TESTS IN ATOMIC SYSTEMS

To consider experiments in atomic systems it is sufficient to restrict the standard-model extension to its QED sector. The modified Dirac equation for a four-component spinor field ψ of mass m_e and charge $q = -|e|$ in an electric potential A^μ is

$$\left(i\gamma^\mu D_\mu - m_e - a_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu - \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu} + ic_{\mu\nu}\gamma^\mu D^\nu + id_{\mu\nu}\gamma_5\gamma^\mu D^\nu\right)\psi = 0 \quad . \quad (4)$$

Natural units with $\hbar = c = 1$ are used, and $iD_\mu \equiv i\partial_\mu - qA_\mu$. The two terms involving the effective coupling constants a_μ and b_μ violate CPT, while the three terms involving $H_{\mu\nu}$, $c_{\mu\nu}$, and $d_{\mu\nu}$ preserve CPT. All five terms break Lorentz invariance. Each particle sector in the standard model has its own set of parameters which we distinguish using superscripts. Since no Lorentz or CPT violation has been observed, these parameters are assumed to be small. A perturbative treatment in the context of relativistic quantum mechanics can then be used. In this approach, all of the perturbations in conventional quantum electrodynamics are identical for particles and antiparticles. However, the interaction hamiltonians including the effects of possible Lorentz and CPT breaking are not the same.

Penning-Trap Experiments

Comparisons of the g factors and charge-to-mass ratios of particles and antiparticles confined within a Penning trap have yielded the CPT bounds in Eqs. (1) and (2). These quantities are obtained through measurements of the anomaly frequency ω_a and the cyclotron frequency ω_c . For example, $g - 2 = 2\omega_a/\omega_c$. These frequencies can be measured to $\sim 10^{-9}$ thereby determining g to $\sim 10^{-12}$.

We have analyzed Penning-trap experiments with electrons and positrons and with protons and antiprotons [11]. We find that to leading order in the Lorentz-violating parameters there are corrections to the anomaly frequencies that are different for particles and antiparticles, while the cyclotron frequencies receive corrections that are the same for particles and antiparticles. Both frequencies have corrections that cause them to exhibit sidereal time variations. We also find that to leading order the g factor has no corrections, and therefore the figure of merit $r_g^e \simeq 0$, even though there is explicit CPT breaking. Because of this, we have proposed using as an alternative figure of merit the relative relativistic energy shifts caused by Lorentz and CPT violation. This is a definition that can be used in any experiment and is consistent with neutral meson experiments, which use mass ratios.

Based on these observations, we suggested looking for two signals of Lorentz and CPT violation: one an instantaneous difference in anomaly frequencies for electrons and positrons, and the other sidereal-time variations in the anomaly frequency of electrons alone. Dehmelt's group at the University of Washington has recently published two papers with the results of these observations [31]. In their first paper, they reanalyzed existing data and obtained a figure of merit $r_{\omega_a}^e \lesssim 1.2 \times 10^{-21}$ from a bound on the difference in the electron and positron anomaly frequencies. In the second paper, they analyzed more recent data for

the electron alone and obtained a bound on sidereal time variations given by $r_{\omega_a^-, \text{diurnal}}^e \lesssim 1.6 \times 10^{-21}$. This corresponds to a bound on the combination of components $\tilde{b}_J^e \equiv b_J^e - m d_{J0}^e - \frac{1}{2} \varepsilon_{JKL} H_{KL}^e$ defined with respect to a nonrotating coordinate system [12] given by $|\tilde{b}_J^e| \lesssim 5 \times 10^{-25} \text{ GeV}$.

Although no $g - 2$ experiments have been made for protons or antiprotons, there have been recent bounds obtained on Lorentz violation in comparisons of cyclotron frequencies of antiprotons and H^- ions confined in a Penning trap [6]. In this case the sensitivity is to the parameters $c_{\mu\nu}^p$, and the figure of merit $r_{\omega_c}^{H^-} \lesssim 10^{-25}$ was obtained.

Clock-Comparison Experiments

The Hughes-Drever type experiments are atomic clock-comparison tests of Lorentz invariance [4]. These experiments look for relative changes between two “clock” frequencies as the Earth rotates. The “clock” frequencies are typically atomic Zeeman or hyperfine transitions. Kostelecký and Lane [12] have made an extensive analysis of these experiments using the standard-model extension. They have obtained approximate bounds on various combinations of the Lorentz-violating parameters from the published results of these experiments. For example, from the experiment of Berglund *et al.* the bounds $\tilde{b}_J^p \sim 10^{-27} \text{ GeV}$, $\tilde{b}_J^n \sim 10^{-30} \text{ GeV}$, and $\tilde{b}_J^e \sim 10^{-27} \text{ GeV}$ have been obtained, respectively, for the proton, neutron, and electron sectors.

Since certain assumptions about the nuclear configurations must be made to extract some of these numerical bounds, these bounds should be viewed as good to within one or two orders of magnitude. To obtain cleaner bounds it is necessary to consider simpler atoms. This is done in the following sections, where it is shown that sharper bounds can be obtained for the proton and electron sectors. However, the clock-comparison tests continue to provide the best bounds for the neutron sector. For example, a recent experiment using a two-species noble-gas maser is consistent with there being no Lorentz or CPT violation in the neutron sector at a level of 10^{-31} GeV [32]. This is currently the sharpest test for the neutron.

Hydrogen-Antihydrogen Experiments

We have investigated the proposed experiments at CERN which will make high-precision spectroscopic measurements of the 1S-2S transitions in hydrogen and antihydrogen [13]. We find that the magnetic field plays a crucial role in the sensitivity of the 1S-2S transition to Lorentz and CPT breaking. For example, in free hydrogen in the absence of a magnetic field, the 1S and 2S levels shift by an equal amount at leading order in hydrogen and antihydrogen. Because of this, there are no leading-order corrections to the 1S-2S transition

frequency in free H or $\bar{\text{H}}$. For hydrogen in a magnetic trap, there are magnetic fields which mix the spin states in the four hyperfine levels. Since the Lorentz-violating couplings are spin-dependent, some of the 1S and 2S levels acquire energy corrections that are not equal. The transitions between these levels have leading-order sensitivity to Lorentz and CPT violation. However, these transitions are field-dependent, making them prone to broadening in an inhomogeneous magnetic field. To be sensitive to leading-order Lorentz and CPT violation in 1S-2S transitions, experiments would have to overcome the difficulties associated with possible line broadening effects due to field inhomogeneities.

We have also considered measurements of the ground-state Zeeman hyperfine transitions in hydrogen and antihydrogen [13]. We find that certain transitions in a hydrogen maser are sensitive to leading-order Lorentz-violating effects. These measurements have now been made by the group of Walsworth at the Harvard-Smithsonian Center using a double-resonance technique [33]. They have obtained bounds on the Lorentz-violation parameters for the proton and electron. The bound for the proton is $|\tilde{b}_J^p| \lesssim 10^{-27}$ GeV. This is an extremely clean bound and is currently the most stringent test of Lorentz and CPT symmetry for the proton.

Spin-Polarized Matter

Experiments at the University of Washington with a spin-polarized torsion pendulum [34] are able to achieve very high sensitivity to Lorentz violation due to the combined effect of a large number of aligned electron spins [14]. The experiment uses stacked toroidal magnets that have a net electron spin $S \simeq 8 \times 10^{22}$, but which have a negligible magnetic field. The apparatus is suspended on a turntable and a time-varying harmonic signal is sought. Our analysis shows that in addition to a signal with the period of the rotating turntable, the effects of Lorentz and CPT violation would induce additional time variations with a sidereal period caused by Earth's rotation. The University of Washington group has analyzed their data and have obtained a bound on the electron parameters equal to $|\tilde{b}_J^e| \lesssim 1.4 \times 10^{-28}$ GeV [35]. This is currently the best Lorentz and CPT bound for the electron.

Muon Experiments

Despite the high precision of recent Lorentz and CPT tests for neutrons, protons, and electrons, it is important to keep in mind that particle sectors in the standard-model extension can be independent of each other and should separately be tested. The situation is similar to CP tests where violation is observed only in the neutral meson sector and not in the lepton or baryon

sectors. A thorough investigation of Lorentz and CPT symmetry should therefore probe as many possible particle sectors as possible. For this reason, we also consider muon experiments, which involve second-generation leptons. We find that there are several different types of experiments that are sensitive to Lorentz and CPT. We have examined both experiments in muonium [36] and $g - 2$ experiments with muons being conducted at Brookhaven [37].

Our results are that experiments measuring the frequencies of ground-state Zeeman hyperfine transitions in muonium in a strong magnetic field are sensitive to Lorentz and CPT violation. If bounds on sidereal time variations are obtained at the 100 Hz level, then the Lorentz-violation parameter for the muon \tilde{b}_J^μ can be bounded at the level of $|\tilde{b}_J^\mu| \leq 5 \times 10^{-22}$ GeV. We also find that in relativistic $g - 2$ experiments using positive muons with “magic” boost parameter $\delta = 29.3$, bounds on Lorentz-violation parameters are possible at a level of 10^{-25} GeV. Experiments looking for sidereal time variations in the muon anomaly frequency would yield stringent new Lorentz and CPT bounds.

SUMMARY AND CONCLUSIONS

By using an extension of QED incorporating Lorentz and CPT violation, we have been able to analyze a variety of atomic experiments. We have shown that low-energy experiments in QED systems are sensitive to suppression factors associated with the Planck scale. We also find that experiments traditionally considered Lorentz tests are sensitive to CPT, and vice versa. The atomic experiments considered here complement those in particle physics and together they are able to test the robustness of the standard model to increasing levels of precision.

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